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Agricultural Productivity and Structural Change: Is Relative Sectoral Price the Mirror Image of Relative Sectoral Productivity?

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Abstract

We have examined the role of sectoral productivity in explaining the process of structural change and relative sectoral prices. Our simple two- sector general equilibrium model demonstrates that an improvement in agricultural productivity can relocate labour away from this sector. Contrary to the conventional wisdom that relative sectoral prices are a mirror image of relative sectoral productivities, we showed a U-shaped relation- ship between them. We estimated our model parameters using the simu- lated method of moments. Our model could largely replicate the farm versus non-farm relative price movement and the agricultural labour share of US economic history.

JEL Classification: F43, O11, O41. Key Words: Agricultural productivity, Structural transformation, manufacturing productivity, Terms

Introduction

The role of agricultural productivity in the process of structural transformation of an economy has been one of the major issues of discussion in the recent literature. The broad consensus is that an improvement in agricultural productivity relocates labour away from the agricultural sector and thereby facilitates the process of structural change. With a constant subsistence consumption of food, less labour is required to produce the same amount of food as agricultural productivity improves. Moreover, Engel's Law states that increased income associated with productivity improvement in agriculture is increasingly spent on industrial goods. Therefore, a non-homothetic preference between agricultural and industrial goods makes a strong case for a positive linkage between labour-saving technical change in agriculture and the movement of labour out of this sector.[1]

Even though the classical works (e.g., Ragnar [1953], Schultz [1953], Rostow [1960]) argued that an improvement in agricultural productivity is essential for industrialisation, the available historical data on industrialisation at the country level tells a different story. Economic historians opine that an improvement in agricultural productivity raises the wage rate making it costly for industry to hire.[2] This scarcity in cheap labor prohibits local industry from flourishing. Historically, Belgium and Switzerland have been less productive in agriculture compared to Netherlands. However, a spectacular growth in the industrial sector happened first in the former two countries and subsequently occurred in the Netherlands (Mokyr [2000]). This negative link between productivity improvement in agriculture and the process of industrialisation is cited by economists as a particular case of the Law of Comparative Advan-

[1] See for example, Murphy, Shleifer and Vishny [1989] Matsuyama [1992], Kongsamut, Rebelo and Xie [2001], Gollin, Parente and Rogerson [2002], Bustos, Caprettini and Ponticelli [2016].

[2] Mokyr [1977], Field [1978], Wright [1979]

tage (Mokyr [1977]). Under this alternative hypothesis, growth of agricultural productivity reduces the price of the agricultural goods, thereby boosting the demand for them. Therefore, the manufacturing sector does not grow.

We have asked the following three questions: **(a)** Is productivity improvement in agriculture essential for the process of industrialisation? **(b)** What is the relationship between agricultural productivity and the relative price (or, terms-of-trade) of manufacturing? Following the literature, when an improvement in agricultural productivity pushes labour out of this sector, we call it a 'Push' channel. The 'Pull' channel is defined when an improvement in manufacturing productivity pulls labour away from the agricultural sector. **(c)** What is an appropriate identification strategy to separate the push channel from the pull channel?

To address question **(a)**, we showed that a productivity improvement in agriculture will push labour out of this sector in countries with a large share of subsistence employment in agriculture (the 'poor' country). This is a robust result in our formulation. However, for those countries where only a small fraction of the population is working in agriculture (the 'rich' country), a productivity improvement in agriculture may or may not release labour from this sector. The movement of labour from agriculture to manufacturing in this latter case depends upon the value of the elasticity of substitution between agriculture and manufacturing goods in the consumer's preference. When the elasticity of substitution is less than unity, the employment share in agriculture will unambiguously decrease due to a faster productivity improvement in agriculture relative to the manufacturing sector. This is famously referred to as *'Baumol's cost disease'* in the literature (following, Baumol [1967]), where the stagnant sector (manufacturing) will attract labour from the progressive sector (agriculture) of an economy despite a rise in both production cost and the prices of the stagnant sector relative to others. We have provided a complete characterisation of the effects of relative sectoral productivity changes in our paper.

To answer question **(b)**, we derived a non-monotonic relationship between productivity improvement in agriculture and the relative price of manufacturing. This result goes contrary to the conventional wisdom that relative prices are a mirror image of relative productivity. On the one hand, any increase in agricultural productivity tends to improve the terms of trade in favour of manufacturing.[3] On the other hand, with improvement in agricultural productivity, labour starts relocating into the manufacturing sector, and this facilitates entry of larger number of intermediate input varieties in the manufacturing sector. Larger intermediate inputs push down the price of the final manufacturing good and this effect dominates when the economy is largely subsistence (or, 'poor') in nature. For an already developed economy, agricultural productivity improvement cannot push a large chunk of labour into manufacturing. A limited entry of intermediate input varieties is not sufficient to push down the relative price of manufacturing. In that case, the relative price of manufacturing rises due to any productivity improvement in agriculture. The overall process gives rise to a U-shaped relationship between agricultural productivity improvement and relative price of manufacturing as the economy starts developing from having large subsistence employment in agriculture ('poor') to becoming a 'rich' economy.

For the empirical part, we constructed the two most important variables of our study from historical US data; these variables are, the relative price and the relative Total Factor Productivity (TFP) between the agricultural and manufacturing sectors during the period 1820-2013.[4] We demonstrated from

[3]This is the so called *Prebisch-Singer hypothesis*. Matsuyama [1992, footnote 8, pp-324] illustrates conditions under which an exogenous growth in agricultural productivity, makes the terms of trade for agriculture deteriorate continuously.

[4]We have interchangeably used the term 'manufacturing' and 'industrial' sector to mean the non-farm sector in this paper.

the data that a one-to-one correspondence between relative price and relative TFP is not true (Figure 4). In fact, two different relative prices for the same relative TFP is a common occurrence in the data. Therefore, as our theory predicted, the non-monotonic relationship between the relative price and the relative TFP is empirically true. We estimated parameters of our model using the methodology of simulated method of moments. Our model generates the structural break observed in the data and matches the evolution of agricultural labour share over time quite well (Figures 6 and 7).

To identify the push and pull channels, as asked in question **(c)**, our model suggests that a faster productivity improvement in agriculture (relative to manufacturing) will push labour out of the agricultural sector when the elasticity of substitution between agriculture and manufacturing goods in consumer preference is less than unity (goods are complements). When that elasticity of substitution is larger than unity (goods are substitutes), a faster productivity improvement is needed in the manufacturing sector to pull labour into this sector. Therefore, we identify push and pull channels through the value of the elasticity of substitution parameter.

These theoretical results of our model provide an *identification strategy* to calibrate the model parameters for the historical US data by judicious use of a structural empirical model. We observed a structural break in the data, sometime during 1920—1965, in the relationship between agricultural labour share and relative sectoral productivity (or, TFP). We have demonstrated this structural break in Figures 4 and 5 by dividing the data into two parts: the former part consists of the years 1820—1919 and the latter part is post-1965. We quantified our observations with a structural empirical model that explains the agricultural labour share using not only the relative sectoral productivity but also the absolute level of agricultural productivity. The estimates of our structural model parameters were found to be different for these two parts of the data, reflecting the visual observation of a structural break.[5] Our demarcation of a structural break in the data along with the formulation of a structural model identifies our model parameters, in particular, the elasticity of substitution which separates the pull channel from the push channel.

Brief Literature Survey

Our paper is complementary to the broad literature on structural transformation and the role of agriculture in it.[6] The theoretical part of our paper is inspired by Matsuyama [1992] who modelled a two-sector economy with non-homothetic preferences. He showed that, an improvement in agricultural productivity leads to an increase in the size of the industrial sector in a closed economy. The larger size of the industrial sector then engenders a higher rate of growth for the economy (due to the presence of a learning-by-doing kind of technological progress). In our model, however, the size of the industrial sector may or may not grow due to productivity improvements in agriculture.[7]

Duranton [1998] showed the non-universality of the linkage between agricultural productivity and industrial development. He modelled an economy with the agriculture and manufacturing sector and incorporated a transport cost. In his paper, industrialisation follows from the progress in the agricultural sector — a result similar to Matsuyama [1992]. We showed both the possibility of industrialisation and the lack of it in our closed economy framework.

[5]We used 1945 as the year of structural break in matching our theoretical model to the data.

[6]See, for example, Matsuyama [1992], Kongsamut, Rebelo and Xie [2001], Gollin, Parente and Rogerson [2002], Caselli [2005], Gollin, Parente and Rogerson [2007], Ngai and Pissarides [2007], Herrendorf and Valentinyi [2012], Herrendorf, Rogerson and Valentinyi [2013], Henderson, Mark and Adam [2013], Jedwab [2013] , Herrendorf, Rogerson and Ãkos Valentinyi [2014], Gollin and Rogerson [2014], Gollin, Jedwab and Vollrath [2016] , Henderson, Adam and Uwe [2017], Alvarez-Cuadrado, Long and Poschke [2017], etc.

[7]Matsuyama [1992] used a *CES* felicity function in appendix B (pp-332) of his paper. In this paper, we have considered a similar felicity function.

Ngai and Pissarides [2007] offered a technological explanation for the mechanism behind structural change in a multi-sectoral model of growth. In their model, sectoral relative prices grow proportionately with the growth in relative sectoral productivities. Therefore, relative prices are related to relative productivities in a one-to-one way. This result is similar to Alvarez-Cuadrado and Poschke [2011] who showed that relative price is uniquely determined by relative productivity. However, the scatter-plot between relative prices and productivities from historical US data of the past two centuries (1820—2013) rejects this monotonic relationship (Figure 4). In fact, relative prices are not uniquely determined by the relative productivities in data and there seems to have been a structural break in this relationship somewhere in between 1920—1965. In our model, sectoral relative prices depend on sectoral productivities as well as sectoral employment share and this explains the empirical observation reasonably well.

In Ngai and Pissarides [2007], employment moves from sectors with higher (TFP) growth to lower growth when preferences are complementary in nature. We obtained similar results in our model in a much simpler setting. In fact, we showed that even when different sectoral TFPs grow at an exactly equal rate, structural change takes place due to non-homotheticity of consumer preference. This is a utility-based explanation of structural change, as both the utility-based and technology-based explanations of structural change co-exist in our paper.

Gollin and Rogerson [2014] considered the issue of transport costs and subsistence agriculture in a closed economic system. They showed that improvement in agricultural productivity, though having an overall negative impact on the share of labor engaged in agriculture (a result similar to Gollin, Parente and Rogerson [2002]), may actually increase the labour share in agriculture in the nearby-city region.[8]

Our paper is related to Dennis and İşcan [2009] and Alvarez-Cuadrado and Poschke [2011]. While the former study contributed to an accounting exercise of structural change based on historical US data, the latter measured the relative importance of 'push' versus 'pull' factors in explaining structural transformation in historical US data (along with data from eleven other industrialised countries). In our paper, we have employed a different empirical strategy for identification of push and pull factors. It may also be noted that Unlike Dennis and İşcan [2009], we do not have access to data on capital. In Section 5.2, we compared our results to those of Alvarez-Cuadrado and Poschke [2011].[9] Our model results highlighted the importance of the 'pull' channel whereas they highlighted both the push and the pull channels as the dominant force behind structural change.

Contribution to Literature

Our paper makes the following contributions to the existing literature: First, we have demonstrated a *non-monotonic relationship* between relative sectoral TFP and relative sectoral price, both in theory and from the US data.[10] Second, we have *identified* a structural break in the relationship not only between relative TFP and relative price, but also between relative TFP and agricultural labour share. Third, we have captured the observation of structural break in a particular structural model constructed using the results of our theoretical

[8] For the relative role of agricultural productivity in the context of urbanization, see, among others, Foster and Rosenzweig [2004, 2007], Jedwab [2013], Henderson, Mark and Adam [2013], Bustos, Caprettini and Ponticelli [2016], and Henderson, Adam and Uwe [2017].

[9] In a recent paper, Alvarez-Cuadrado, Long and Poschke [2017] incorporate capital in a two sector Solow-type growth model and show that differences in the sectoral elasticity of substitution between capital and labour will reallocate labour away from the agricultural sector, thereby inducing structural change.

[10] Alvarez-Cuadrado and Poschke [2011] provided historical data for 12 developed countries with a present employment share in agriculture of less than ten percent. In most of these cases, the relative price of manufacturing evolved in a U-shaped pattern over time.

model. We appealed to the *Simulated Method of Moments* in conjunction with this structural model to estimate the values of the critical parameters of our model. One of our model parameters, the parameter for elasticity of substitution, offers an identification strategy between the push and the pull channels. Fourth, when we estimated parameters of our model to match the period-wise agricultural labour share from 1820 to 2013, our model performed reasonably well. Fifth, we have demonstrated through a counterfactual, the importance of falsifiable model building (Popper [2005]) regarding the relationship between relative price and relative TFP in identifying the push and pull channels.

The rest of our paper is organised as follows. Section (2) lays down our basic model. Comparative static results are provided in section (3). Section (4) describes the methodology of our estimation along with description of the empirical patterns. Section (5) discusses the quantitative implications of our estimates to the historical data of the US economy on productivity and prices. Finally, section (6) concludes the paper.

1 The Economic Environment

1.1 Preliminaries

In the economy, there is a continuum of agents of measure L, each endowed with one unit of labor. The economy has two sectors — agriculture and manufacturing. Labour is freely mobile across these two sectors, equalising the wage rate between them. A unit of labour receives a competitive wage denoted by w. The market structure is perfectly competitive in the agriculture sector. Moreover, perfect competition describes the market structure of the final goods' production in the manufacturing sector, while intermediate goods' production in the manufacturing sector is characterised by monopolistic competition.

A representative agent's utility maximisation problem is given by

$$\underset{\{c_A, c_i\}}{\text{Max}} \quad U = \left[b(c_A - \gamma)^\theta + c_M^\theta \right]^{\frac{1}{\theta}}; \quad c_A > \gamma > 0, \quad \theta \in (-\infty, 1) \tag{1}$$

$$\text{subject to} \qquad p_A c_A + p_M c_M = w, \tag{2}$$

where c_A and c_M are the consumption levels of the agricultural and the manufacturing goods, respectively. The subsistence level of consumption of the agricultural good is denoted by the parameter γ. The relative bias for consumption of the agricultural good as opposed to the manufacturing good is denoted by the parameter b (> 0). Wage income, denoted by w, is the only source of income for the agents in our model. We define $E \equiv \frac{1}{1-\theta} \in (0, \infty)$ as the elasticity of substitution between the two goods in the agent's preference.

We normalise the price of the agricultural good to unity: $p_A \equiv 1$. With this normalisation, p_M represents the relative price of the manufacturing good, which is alternatively called the manufacturing terms-of-trade. The utility maximisation problem yields the following first order condition:

$$c_A = \gamma + c_M \left(b \cdot p_M \right)^E.$$

Multiplying both sides of the above condition by L results in the following equation:

$$L \cdot c_A = L \cdot \gamma + L \cdot c_M \left(b \cdot p_M \right)^E.$$

Let us denote the aggregate production of the agricultural good by x_A and that of the manufacturing good by x_M. Then, market clearing conditions equating demand to supply (i.e., $L \cdot c_A = x_A$ and $L \cdot c_M = x_M$) are translated into the following equation:

$$x_A = L\gamma + x_M \left(b \cdot p_M \right)^E. \tag{3}$$

The term $L\gamma$ in eq. (3) represents the aggregate subsistence consumption of food in the economy.

1.2 Production

Labour is the only factor of production in the economy. We assume that the agricultural good is produced by the following linear production function,

$$x_A = A \cdot L_A. \tag{4}$$

L_A is the amount of labour required to produce x_A amount of agricultural goods and A is the measure of agricultural productivity. With aggregate subsistence consumption given by $L\gamma$, $\frac{L\gamma}{A}$ workers are required to be engaged in the subsistence production within the agricultural sector. We assume that the agriculture sector is sufficiently productive so that the following inequality always holds true:

$$L > \frac{L\gamma}{A} \text{ or, } A > \gamma. \tag{5}$$

Note that, in an economy of size L, only $\frac{L\gamma}{A}$ workers are engaged in the subsistence sector. Here, we define the term $L_S \equiv \frac{L\gamma}{A}$ as the subsistence level of employment in the economy. The fraction of subsistence employment in the economy is free from any scale effect as $\frac{L_S}{L} = \frac{\gamma}{A}$. In an economy where A is comparatively lower, a larger fraction of the work force is engaged in subsistence production, while the opposite is true in an economy with higher agricultural productivity.[11]

Since the agricultural sector is perfectly competitive, the wage is given by the marginal productivity. Our production technology in eq. (4), along with the normalisation of agricultural price to unity implies that the wage rate is

[11] See Gollin and Rogerson [2014] for empirical evidence on association between high employment in subsistence agriculture and low agricultural productivity in sub-Saharan African economies.

determined by the agricultural productivity parameter. In particular,

$$w = A. \tag{6}$$

With free mobility of workers across sectors, the same wage rate is applicable to all the labourers, which is the only income for the agents in this economy.

The production of the manufacturing goods requires n number of differentiated intermediate inputs. These inputs are aggregated using a CES technology to produce the final manufacturing good, as given in the production function below:

$$x_M = M \left(\sum_{i=1}^{n} z_i^{\delta} \right)^{\frac{1}{\delta}}; \quad \delta \in (0, 1), \tag{7}$$

where z_i is the amount of i^{th} intermediate input used in the production of the final good. We define the elasticity of substitution between any two intermediate inputs as $\sigma \equiv \frac{1}{1-\delta} > 1$. The parameter M captures the productivity in the manufacturing sector. For higher values of M, the same amount of intermediate inputs produce more of the final goods.

We assume that $\sigma \geq E$, i.e., the elasticity of substitution among different varieties of intermediate inputs in the production of the final good is larger than is the elasticity of substitution between the respective consumption of agricultural and manufacturing goods.[12]

The production of the final good is done under perfect competition. Let π_M denote the profit and p_i be the price per unit of i^{th} intermediate input. Then

[12] If we had allowed the industrial goods in the utility function to be an aggregate consumption index of n different final good varieties, then the assumption $\sigma \geq E$ would simply mean that industrial goods are more substitutable among themselves in consumption than they are, in general, with the agricultural good. As an example, it makes sense to assume that two varieties of car are more substitutable with each other than a can would be, in general, with either rice or wheat. None of our results would change with such a modification by allowing intermediate inputs to be regarded as consumption varieties in the utility function. However, we choose to work with intermediate input varieties and assume that $\sigma \geq E$.

profit maximisation in the final manufacturing good sector can be given by

$$\max_{z_i \geq 0} \pi_M = p_M x_M - \sum_{i=1}^{n} p_i z_i; \quad \text{subject to } eq. \text{ (7)}.$$

By solving this profit maximisation, we obtain the following demand function for the intermediate input,

$$z_i = \frac{p_i^{-\sigma} \left(\sum_{j=1}^{n} z_j p_j \right)}{\sum_{j=1}^{n} p_j^{1-\sigma}}; \quad \forall i \in [1, n]. \tag{8}$$

The above demand function, along with the condition that profit must be zero (i.e., $\pi_M = 0$) under perfect competition, ensures that the price of the final manufacturing good becomes

$$p_M = \frac{1}{M} \left(\sum_{j=1}^{n} p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{9}$$

Note that an exogenous improvement in the manufacturing productivity parameter, M, leads to a decrease in the price of the final manufacturing good. Also an increase in the number of intermediate inputs, denoted by n, leads to an efficiency gain in the manufacturing sector. This gain is purely due to the specialisation effect — growth in the number of inputs, each being more specialised leads to an overall efficiency gain in the production process.

1.3 Intermediate inputs

Each variety of the intermediate inputs is being produced by a monopoly producer. The production of variety i needs both fixed cost, denoted by α, and a constant marginal cost, denoted by β. The production function of i^{th} intermediate good is given by

$$L_i = \alpha + \beta z_i,$$

where L_i is the amount of labour hired by i^{th} producer. The producer of the i^{th} variety intermediate input faces the following profit maximisation problem;

$$\max_{p_i} \pi_{z_i} = p_i z_i - (\alpha + \beta z_i) w.$$

subject to the demand function for her product, z_i, given in eq. (8). While maximising profit, each producer takes p_M as given even though it depends on the choice of p_i (see eq. (9)). The profit maximisation problem along with free entry in the intermediate input production sector gives the following solutions of price and quantity.

$$p_i = \frac{A\beta}{\delta}, \quad \text{[using } w = A\text{, by eq.(6)]};$$
$$z_i = \frac{\alpha\delta}{(1-\delta)\beta}; \quad \forall i = 1, 2, \ldots, n.$$

Aggregate employment in the intermediate goods sector which is equivalent to manufacturing employment is denoted by L_M and is given by

$$L_M = \sum_{i=1}^{n} L_i = \frac{n\alpha}{1-\delta}. \tag{10}$$

With these solutions, aggregate production of the final manufacturing good, in eq. (7), and the price index, in eq. (9), takes the following form,

$$x_M = M \frac{\alpha\delta}{(1-\delta)\beta} n^{\frac{1}{\delta}}; \tag{11}$$
$$p_M = \frac{A\beta}{M\delta} n^{-\left(\frac{1}{\delta}-1\right)};. \tag{12}$$

Note that, labour is not directly employed in the final manufacturing good production [see eq. (7)]. However, they are employed indirectly through intermediate goods' production. To see this, let us re-write eq. (11) using (10), as follows

$$x_M = L_M \frac{M\delta}{\beta} n^{\frac{1}{\delta}-1}.$$

Then, the marginal productivity of labour in the final manufacturing goods' sector is given by

$$\frac{\partial x_M}{\partial L_M} = \frac{M\delta}{\beta} n^{\frac{1}{\delta}-1}. \tag{13}$$

By multiplying the above expression by p_M [as given in eq. (12)], we derived the value marginal productivity in the manufacturing sector, which must be equal to the wage rate, A. This verification guarantees that the wage rate is equalised across sectors.

Note that the marginal productivity expression in eq. (13) is increasing in n. From eq.(10), we determined that n rises monotonically with the size of the manufacturing sector. Thus, larger size of the manufacturing sector is associated with higher marginal productivity of its workers.[13] Therefore, given a particular wage, an increase in size of the manufacturing sector lowers the (relative) price of manufacturing.[14] .

Finally the labour market clearing condition ensures that

$$L_A + L_M = L.$$

Using eqs. (4) and (10), the above equation can be re-written as

$$x_A = AL - \frac{An\alpha}{1-\delta}. \tag{14}$$

Next, using equations (11), (12), and (14) and the definitions of E and σ, we can

[13]This is in contrast to the standard neo-classical production function, where, due to diminishing marginal productivity assumption, larger size of a sector reduces the marginal productivity of its workers.

[14]To see this, note that $w = p_M \times$ (marginal productivity). Since w is fixed at A, an increase in marginal productivity must reduce p_M.

re-write eq. (3) as follows:

$$L - n\alpha\sigma = \frac{L\gamma}{A} + n^{\frac{\sigma-E}{\sigma-1}} \left(\frac{A}{M}\right)^{E-1} \alpha\beta^{E-1} b^{E}\sigma^{E}(\sigma-1)^{1-E}$$

This equation solves for n uniquely, as established by the following lemma:

Lemma 1 (**Existence and uniqueness of equilibrium**). *There exists a unique equilibrium solution of n from equation* (15).

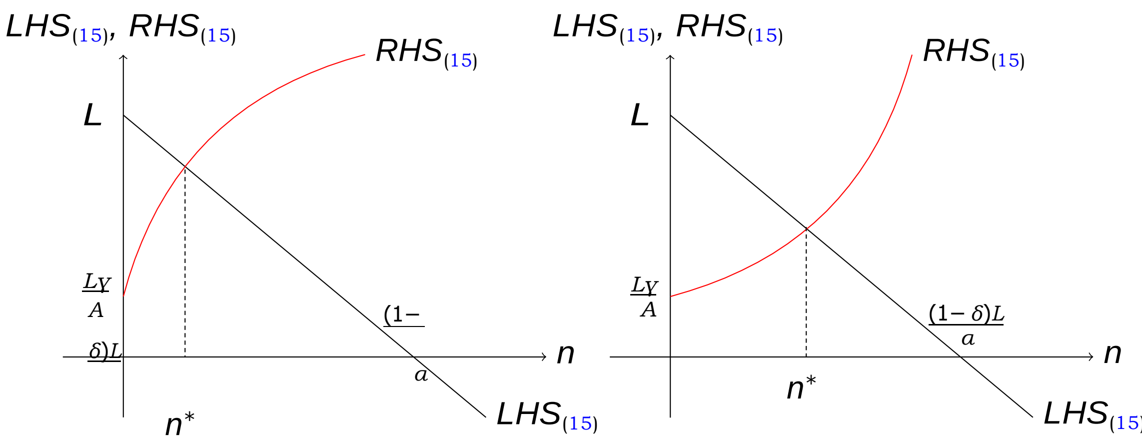
Proof: Define the left-hand side of eq.(15) as

$$LHS_{(15)} \equiv L - n\alpha\sigma;$$

and the right-hand side as

$$RHS_{(15)} \equiv \frac{L\gamma}{A} + n^{\frac{\sigma-E}{\sigma-1}} \left(\frac{A}{M}\right)^{E-1} \alpha\beta^{E-1} b^{E}\sigma^{E}(\sigma-1)^{1-E}.$$

Clearly, $LHS_{(15)}$ is a monotonically decreasing function of n. The expression $RHS_{(15)}$ is an increasing function of n, since $\sigma \geq max\{1, E\}$. $RHS_{(15)}$ is a concave function of n for the case $E > 1$ (panel (a) of figure 1) and convex function of n for $E < 1$ (panel (b) of figure 1). It becomes a linear function of n for the case $E = 1$. The expression $RHS_{(15)}$ takes the value $\frac{L\gamma}{A}$ at $n = 0$ and it approaches infinity as n approaches infinity. Similarly, note that $\frac{L\gamma}{A} < L$ by equation (5). Then, we must have $LHS_{(15)} = RHS_{(15)}$ at some $n = n^*$. This, proves the *existence* of a solution of n. This solution must also be *unique*, as the difference $[LHS_{(15)} - RHS_{(15)}]$ is a monotonically decreasing function of n. □



(a): Concave RHS case: $E > 1$ (b): Convex RHS case: $E < 1$

Figure 1: Existence and uniqueness of equilibrium number of input varieties.

3 Comparative statics

For comparative statics exercises, let us define the percentage change of any variable, namely x, as $\hat{x} = \frac{dx}{x}$. Using eq.(15), taking logarithm in both sides, performing total derivatives and rearranging expressions, we obtained the following equation:

$$\hat{n}\left[\frac{n\alpha\sigma}{L - n\alpha\sigma - \frac{L_Y}{A}} + \frac{\sigma - E}{\sigma - 1}\right] = -(E-1)\left[\frac{\frac{L_Y}{A}}{L - n\alpha\sigma - \frac{L_Y}{A}}\right]\hat{A} + (E-1)\hat{M}. \quad (16)$$

Similarly, using eq.(12), we derived the following expression:

$$\hat{p_M} = \hat{A} - \hat{M} - \left(\frac{1}{\delta} - 1\right)\hat{n}. \quad (17)$$

The exogenous variables are A and M and the endogenous variables are n and p_M. Note that, due to the introduction of endogenous product variety, part of the productivity growth is endogenous to the manufacturing sector. However, by our measure of productivity growth we have referred to the growth in the exogenous productivity parameter only.

In the literature, the occurrence of structural change has two competing explanations: technological and utility-based. In the technological explanation, structural change takes place due to differing rates of sectoral factor productivity growth (e.g., Ngai and Pissarides [2007]). In utility-based approach, different income elasticities can generate structural change even when sectoral productivities grow at equal rates (e.g., Gollin, Parente and Rogerson [2002], Alvarez-Cuadrado and Poschke [2011]). In our model, both these explanations can co-exist. For technological explanations, set $\gamma = 0$ in eq.(16) and see that $\hat{n} \neq 0$ as long as $\hat{A} \neq \hat{M}$. Similarly, for the utility-based approach, set $\gamma \neq 0$ and see that $\hat{n} \neq 0$ even when $\hat{A} = \hat{M}$. We analysed the effects of changes in productivity on sectoral relocation of labour and relative prices.

3.1 Agricultural productivity changes

With improvement in agricultural productivity only, we put $\hat{A} > 0$ and $\hat{M} = 0$ in equations (16) and (17). From there, we derived the following set of equations:

$$\hat{n}\left[\frac{L_M}{L_A - \frac{L_Y}{A}} + \frac{\sigma - E}{\sigma - 1}\right] = -\left[E - 1 - \frac{\frac{L_Y}{A}}{L_A - \frac{L_Y}{A}}\right]\hat{A}. \tag{18}$$

and

$$\hat{p}_M = \hat{A}\,\frac{L\left(1 - \frac{\sigma}{\sigma - 1}\frac{Y}{A}\right)}{L_M + \frac{\sigma - E}{\sigma - 1}\left(L_A - \frac{L_Y}{A}\right)} \tag{19}$$

In deriving the above two equations, we used the fact that $n\alpha\sigma = L_M$ and $L - L_M = L_A$. As is evident from eq. (18), the value of E is crucial in determining the effect of changes of A on n. Similarly, it can be seen from eq. (19) that the values of σ and $\frac{Y}{A}$ become crucial in determining the effects of changes of A on

P_M.

We first analysed the case where agriculture and manufacturing goods are gross complement in preferences (i.e., $E \in (0,1)$). From eq.(18), it is clear that the sign of the coefficients of $\hat{n}$ and $\hat{A}$ are the same when $E \in (0,1)$. This implies that an improvement in the agricultural productivity must raise the share of employment of manufacturing. This last result follows, as the number of intermediate product varieties is directly related to the manufacturing employment share (see eq.(10)). To put it differently, an improvement in the agricultural productivity lowers the employment share in agriculture.

To see the effects of change in A on the manufacturing terms of trade, we used eq.(19). Here the sign of the coefficient of $\hat{p_M}$ crucially depends on the sign of the term $\left(1 - \frac{\sigma}{\sigma-1}\frac{Y}{A}\right)$. It can be seen that, for an improvement in agricultural productivity, the manufacturing terms of trade will fall if $\frac{Y}{A} > 1 - \frac{1}{\sigma}$. We already interpreted the fraction $\frac{Y}{A}$ as the share of the subsistence employment in aggregate employment. Thus, when the subsistence employment share is relatively high (as happens in "poor" economies), agricultural productivity improvement tends to depress the manufacturing terms of trade. With a gradual increase in A, as soon as the condition $\frac{Y}{A} < 1 - \frac{1}{\sigma}$ is satisfied, the manufacturing terms of trade starts to improve. This gives rise to an inverted U-shaped relationship between the relative price of manufacturing and agricultural productivity.

To see the effects of A on n in the gross substitute case (i.e., $E > 1$), we re-wrote eq. (18) as below:

$$\hat{n} = -\hat{A}(\sigma-1)\frac{(E-1)L_A - EL_S}{(\sigma-1)L_M + (\sigma-E)(L_A-L_S)}. \tag{20}$$

Note that we previously defined L_S as the subsistence level of employment in the economy. Using the above equation, and for $\hat{A} > 0$, we obtained

$$\hat{n} < (=) > 0 \quad \text{if and only f} \quad \frac{L_S}{L_A} < (=) > \underline{E-1}\;.$$

E

With gross substitutability in preferences ($E > 1$), an improvement in agricultural productivity leads to a decrease in manufacturing employment ($\hat{n} < 0$), as long as the share of the subsistence sector in total agricultural employment is less than a critical level given by $\frac{E-1}{E}$.

For many underdeveloped countries, the size of subsistence employment within agriculture is very high. In many African countries, almost 50% of the entire work force is engaged in subsistence agricultural employment. In Uganda for example, as many as 43% of all working persons were engaged in subsistence agriculture — according to reports of the Uganda Bureau of Statistics in 2014.[15] The condition $L_S > \frac{E-1}{E} L_A$ is likely to be satisfied for the least developed countries. Thus, improvement in agricultural productivity should reduce the size of the agricultural sector in poor countries, even accounting for a large degree of substitutability in preferences.

We summarised these results in the following proposition:

Proposition 1. *Assume that preferences are given as in equation (1). An improvement in agricultural productivity leads to*

(i) a decline in the share of employment in the agriculture if there is gross complementarity in the preferences.

(ii) a decline in the share of employment in the agriculture if there is gross substitutability in the preferences and $\frac{L_S}{L_A} > \frac{E-1}{E}$.

(iii) a decline in the manufacturing terms of trade for lower values of A and an increase in the same for higher values of A. This gives rise to a U-shaped curve between manufacturing terms of trade and agricultural productivity irrespective of the value of elasticity of substitution in preferences.

[15] See pp. 21 of the Uganda Bureau of Statistics, http://www.ubos.org/onlinefiles/uploads/ubos/statistical_abstracts/Statistical_Abstract_2014.pdf. Agricultural employment share is 71% in Uganda. Then, the ratio $L_S/L_A = 0.61$ implies that, for all $E \leq 2.56$, $\hat{n} > 0$ as long as $\hat{A} > 0$.

3.2 Manufacturing productivity changes

With a change in manufacturing productivity only, we used equation (16) to get

$$\hat{n}\left(\frac{n\alpha\sigma}{L_A - L_S} + \frac{\sigma - E}{o - 1}\right) = (E - 1)\hat{M}; \tag{21}$$

and used equation (17) to get

$$\hat{p_M} = -\hat{M}\left(\frac{L - L_S}{L_M + \frac{}{\sigma - 1}(L_A - L_S)}\right) \tag{22}$$

From eq.(22), as manufacturing productivity improves, the terms of trade of manufacturing decline, irrespective of the value of the elasticity of substitution. However, from eq.(21), the inter-sectoral relocation of labour now crucially depends on the value of the elasticity of substitution parameter, E. Improvement of manufacturing productivity pulls labour toward the manufacturing sector if $E > 1$ and pushes labour out of manufacturing sector if $E < 1$. For the case where $E = 1$ (Cobb–Douglas preferences), manufacturing productivity changes do not affect the inter-sectoral labour reallocation.

We summarised these results in the following proposition:

Proposition 2. *Suppose preferences are given as in equation (1). An improvement in manufacturing productivity leads to*
(i) a decline (an increase) in the share of employment in the agricultural sector if $E > 1$ ($E < 1$). With $E = 1$, inter-sectoral labour allocation becomes independent of any change in manufacturing productivity.
(ii) a decline in the manufacturing terms of trade.

The result that the relative price of the manufacturing goods declines with productivity improvement in manufacturing is intuitive. Note that an increase in productivity raises the marginal (as well as the average) productivity of labour in the manufacturing sector. With a competitive market structure, manufacturing workers are paid according to their value marginal product — i.e., $p_M \cdot MP_L^M = A$, where MP_L^M is the marginal product of labour in the manufacturing sector (see equation (13)) and A is the wage rate. As M increases, it raises the MP_L^M and hence, p_M must fall since the wage is constant. When the size of the manufacturing sector grows with M , the relative price falls at a faster rate, both due to the direct negative effect of M on p_M and through the indirect negative effect of a larger n on p_M. However, when n falls with an improvement in M (with $E < 1$), this indirect effect mitigates the fall in p_M but cannot change the direction of its (downward) movement. Thus, relative price falls (albeit, at a slower rate) with an increase in M in this case, with $E < 1$.

The relationship between the size of the manufacturing sector and its productivity crucially depends on the elasticity of substitution parameter (as in proposition 2(i)). When $E < 1$, goods are complements in the preferences. In this scenario, a decrease in the price of manufacturing goods (following an improvement in manufacturing productivity) leads to an increase in the demand for the agricultural goods. This requires more production of the agricultural good. So, labour moves away from industry to join the agricultural sector. In fact, the expenditure share of the manufacturing sector (in GDP) also declines when $E < 1$.[16]

Similarly, one can explain the case with $E > 1$. Here goods are substitutes. With a decrease in p_M (induced by manufacturing productivity improvement), demand for agricultural goods goes down. This is because, with substitutability, people move toward relatively cheaper (manufacturing) goods. In the pro-

[16]Expenditure share of manufacturing in GDP is given by $\frac{p_M x_M}{AL}$. Here, aggregate consumption expenditure of the manufacturing goods is denoted as $p_M x_M$ and aggregate GDP is AL, which is the national income in this model. Using equations (10), (11) and (12), we can express this expenditure share as $\frac{p_M x_M}{AL} = \frac{L_M}{L}$. Similarly, expenditure share of the agricultural sector in national income is given by $\frac{x_A}{AL} = \frac{L_A}{L}$. Thus, sectoral employment shares exactly reproduce sectoral GDP shares. This is broadly in line with the empirical facts about the relationship between sectoral employment and expenditure shares — see Herrendorf, Rogerson and Ãkos Valentinyi [2014] for detailed evidence of this.

cess, labourers are released from agriculture and find their way into manufacturing. On net, the size of the manufacturing sector goes up along with its expenditure share in GDP as long as $E > 1$.

3.3 Relative productivity changes

To analyse the effect of relative productivity change on employment share and relative prices, we re-wrote equations (16) and (17) as follows:

$$\hat{n} = (\hat{M} - \hat{A})\frac{E-1}{k_1} + \hat{A}\frac{k_2}{k_1}; \tag{23}$$

$$\hat{p_M} = -(\hat{M} - \hat{A})\left(1 + \frac{1}{\sigma - 1}\frac{E-1}{k_1}\right) - \hat{A}\frac{1}{\sigma - 1}\frac{k_2}{k_1}. \tag{24}$$

The expressions of k_1 and k_2 take the following form,

$$k_1 = \frac{L_M}{L_A - L_S} + \frac{\sigma - E}{\sigma - 1} > 0 \quad \text{and} \quad k_2 = \frac{L_S}{L_A - L_S} > 0.$$

From eq.(23), one can find that when $E < 1$, faster productivity improvement in agriculture relative to manufacturing, i.e., $\hat{A} > \hat{M} > 0$, will always lower the employment share in agriculture. Similarly, when $E \geq 1$, faster productivity improvement in manufacturing relative to agriculture, i.e., $\hat{M} > \hat{A} > 0$, will always lower the employment share in agriculture.

From eq.(24), we can easily show that the term $\left(1 + \frac{1}{\sigma-1}\frac{E-1}{k_1}\right)$ is always positive for all values of $E \in (0, \infty)$. Then a faster productivity improvement in manufacturing relative to agriculture will always lower the manufacturing terms of trade.

One interesting case is an equal proportionate increase in the productivity of agriculture and manufacturing, i.e., $\hat{M} = \hat{A} > 0$. In this case, both the employment share in agriculture and the manufacturing terms of trade will unambiguously decrease due to productivity improvement.

We summarised these results in the following proposition:

Proposition 3. *Suppose that preferences are given as in eq.* (1) *and that both agricultural and manufacturing productivity increase. The employment share in agriculture will unambiguously decrease due to - (i) (* ***push channel****) a faster productivity improvement in agriculture relative to manufacturing when goods are complementary in preferences (i.e.,* $E < 1$*); (ii) (* ***pull channel****) a faster productivity improvement in manufacturing relative to agriculture when goods are substitutable in preferences (i.e.,* $E > 1$*); and (iii) an equal proportionate increase in agricultural and manufacturing productivity.*
The relative price of manufacturing will unambiguously decrease due to - (iv) a faster productivity improvement in manufacturing relative to agriculture; and (v) an equal proportionate increase in agricultural and manufacturing productivity. For faster productivity improvement in agriculture relative to manufacturing, movement in the relative price of manufacturing is ambiguous.

These results can easily be established using eqs. (23) and (24). Note that, result (i) in proposition 3 establishes that employment share moves from the sector with the higher productivity growth (agriculture) to the sector with lower productivity growth (manufacturing) when $E < 1$. The exactly opposite happens when $E > 1$ as shown in the results in proposition 3(ii). Then, the higher productivity growth sector (manufacturing) attracts employment from the lower productivity growth sector.

The results in proposition 3 are similar to those in Ngai and Pissarides [2007] but were derived in a much simpler setting.[17] Our result in the $E < 1$ case confirms the facts of structural change as identified by Baumol, Blackman and Wolff [1985]. With price inelasticity of demand, sectors with lower productivity growth rate attract employment from elsewhere. This may happen

[17] See proposition 2, pp-433 in Ngai and Pissarides [2007].

despite the rise in their relative price. This is often referred to as 'Baumol's cost disease'.[18]

In the section of our paper, we identified a push versus pull channel through the values of E, as shown in proposition 3. Particularly, we showed that in the data, labour share in agriculture has been falling continuously. At the same time, relative productivity in agriculture has been rising during the period 1820—1965, while relative productivity in manufacturing has been rising during the period 1966-2013. Therefore, to explain the falling labour share in agriculture, it must be the case that $E < 1$ during the period 1820–1965, and $E > 1$ since then. Our empirical results mostly match this identification strategy (see section 5.2).

4 Structural Break, Structural Model, and Estimation Methodology

4.1 Observing Structural Break: Productivity and Price in the United States

We followed Alvarez-Cuadrado and Poschke [2011] in constructing the historical data series. In particular, we constructed the following three historical series from the US data: (1) the relative price of the manufacturing sector output with respect to the agricultural sector output using the producer price index data, (2) TFP for the agricultural sector and (3) TFP for the manufacturing sector. Figure 2 illustrates the evolution of both the agricultural sector and the manufacturing sector TFPs. We have normalised the 1820 TFP of both the

[18] Baumol [1967] claimed that the stagnant sector will attract labour from progressive sectors of an economy despite rise in production cost and prices of the stagnant sector relative to others. For more discussion on this issue see Ngai and Pissarides [2007] (footnote-1, pp-430).

sectors to 100. Over the last two centuries, growth of the manufacturing TFP has been almost twice that of the agricultural TFP.

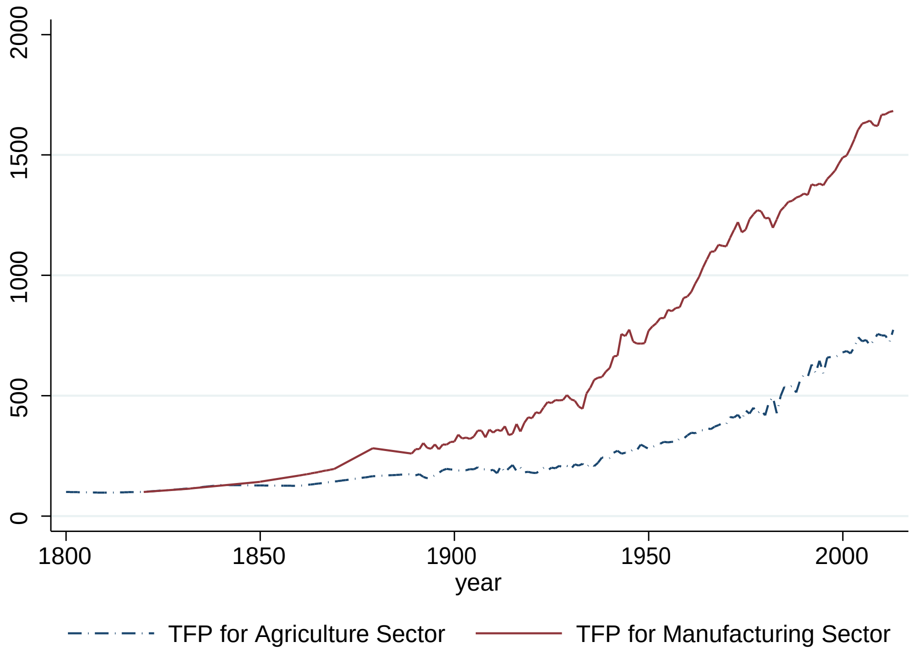


Figure 2: Total Factor Productivity for the agricultural and manufacturing sectors. We have normalised the 1820 TFP of both the sectors to 100. [***Source:*** *The farm productivity is gathered various sources: from Gallman [1972] for 1800–1840, from [Craig and Weiss, 2000, Table 3] for 1840–1870, from Kendrick [1961] for 1869–1948, and from the United States Department of Agriculture (USDA) Economic Research Service, Agricultural Productivity Dataset, http://www.ers.usda.gov/Data/AgProductivity/, for 1948–2013. Nonfarm productivity is from Sokoloff [1986] for 1820–1860, from Kendrick [1961] for 1870–1948, and from the Bureau of Labor Statistics (BLS) Multifactor Productivity Trends—Historical SIC Measures 1948–2013, http://www.bls.gov/mfp/historicalsic.htm, for 1948–2013.]*

The evolution of the relative TFP defined as the manufacturing sector TFP over the agricultural sector TFP, is illustrated in Figure 3 along with the relative sectoral price. Initially the relative price of manufacturing declined until about 1940, with some ups and down in between. It has shown an upward trend since 1940. Overall, this resembles a U-shaped curve. The relative TFP graph, however, resembles an inverted U-shaped curve.

We plotted the relative price against the relative TFP in the logarithmic

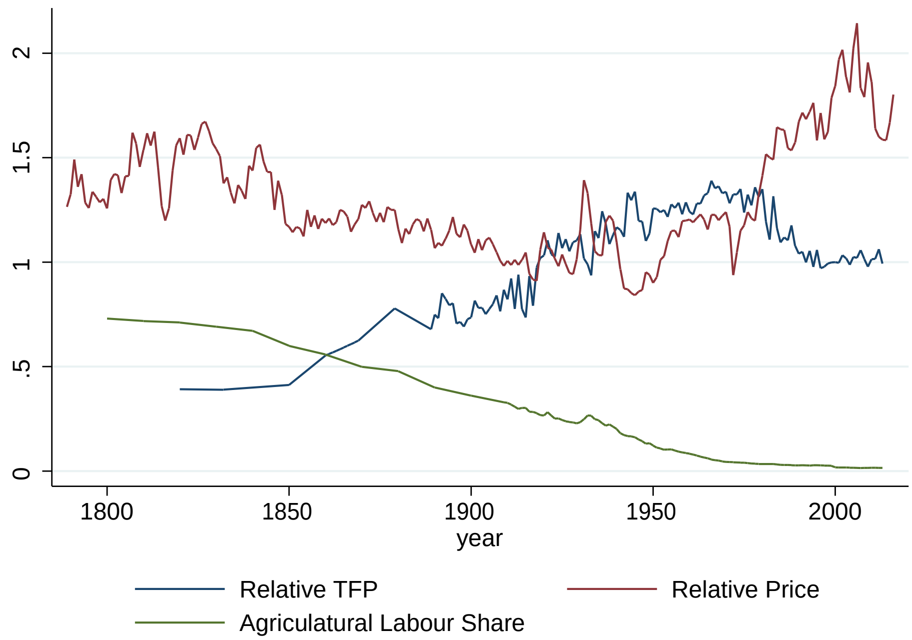


Figure 3: Relative TFP ($\equiv \frac{TFP_M}{TFP_A}$) and Relative Price ($\equiv \frac{p_M}{p_A}$) for agricultural and manufacturing sectors during the period 1820-2013. Also the agricultural labour share for the same period has been plotted.

scale in Figure 4. The relative price is not uniquely determined by the relative TFP, i.e., a one-to-one relationship between the relative price and the relative TFP may be ruled out. Therefore, some other exogenous variable, besides the relative TFP, may determine the relative sectoral price. It can be seen from Figure 4 that the relative price is much less responsive to changes in the relative TFP during the pre-1920 period than during the post-1965 period. The relationship between these variables is not so clear during the intermediate period of 1920—1965. It indicates that there has been a structural break in the data somewhere during 1920—1965.

The plot of the share of agricultural labour against the relative TFP in figure 5 demonstrates that, for pre-1920 data points, the agricultural labour share falls with an increase in the relative TFP of manufacturing. However, this

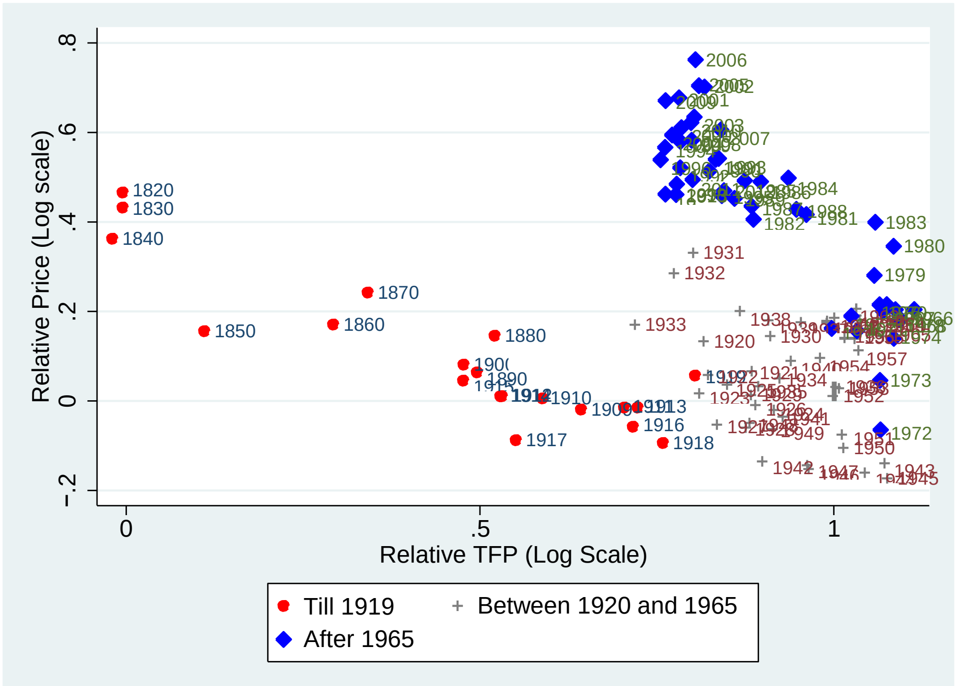


Figure 4: Relative price plotted against relative TFP in logarithmic scale. The plot demonstrates that multiple relative prices have been observed for the same relative TFP over time.

relationship is reversed for the post-1965 data points, where the agricultural labour share increases with an increase in the relative TFP of manufacturing. As earlier, this observation is suggestive of some other exogenous variable, besides the relative TFP, as being a determinant of the relative price. Also, it is clear that, somewhere during 1920—1965, there has been a structural break in the data between the relative TFP and the agricultural labour share. Without loss of generality, we considered 1945 as the year of the structural break in our empirical model.

4.2 Formulation of a Structural Model

In the theoretical part, we observed that relative sectoral price and agricultural labour share are non-monotonic functions of relative TFP. We used them to

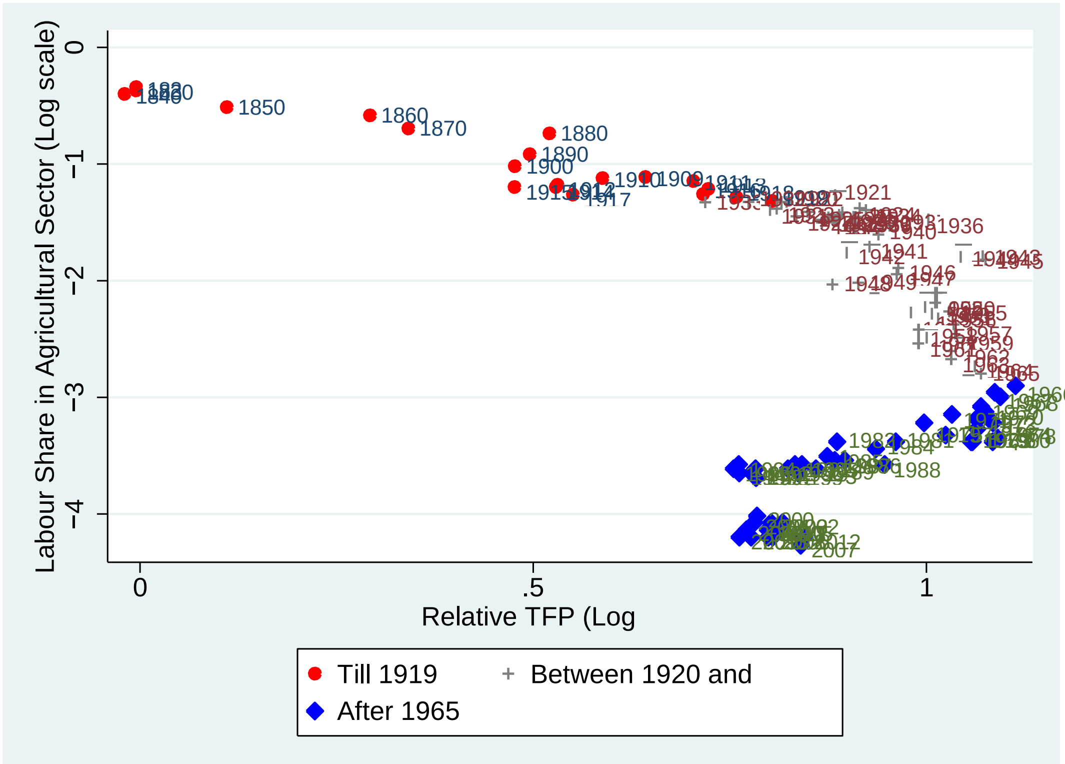


Figure 5: Agricultural Labour Share plotted against relative TFP in logarithmic scale. The plot demonstrates a negative relationship between the variables during 1820-1919 compared to a positive one during post-1965.

create a structural model for the empirical study. In particular, we used eq. (15), derived in section 2, by replacing n in terms of L_A using eq. (10) and the full employment condition, $L_M + L_A = 1$. From eq. (10), we found that $n = \frac{1-}{\alpha\sigma}$. Using this, we rewrote eq. (15) in terms of agricultural labour share only as follows.

$$L_A = \frac{\gamma}{A + (1 - L_A)^{\sigma-1} \left(\frac{A}{M}\right)^{\frac{E-1}{\sigma-1}} \alpha^{\frac{-1}{\sigma-1}} \sigma^{\frac{\sigma(-1)}{\sigma-1}} (\sigma - 1)\, \beta^{1-E}\, b^{E-1} E} \tag{25}$$

From eq. (25) , L_A can be solved uniquely in terms of A and $\frac{A}{M}$. This was already demonstrated in Section 2 (through the solution of n in figure 1). In essence, our theoretical model predicts that relative price and the agricultural labour share will depend on the agricultural TFP along with the relative TFP. The logarithmic version of eq. (25) is:

$$\log L_A \quad = \quad K_1 + K_2 \log A + K_3 \log \frac{A}{M} + \upsilon_2 \tag{26}$$

In the above equation, the underlying variables, such as $\frac{A}{M}$, A, and L_A are all non-stationary. The regression described by eq. (26) represents a textbook case of spurious correlation indicated by high values of R^2. Therefore, the consistency and applicability of the estimates are in doubt. To address these concerns, we proceeded as follows. Let Δ denote the difference operator that represents change over two successive time points in the data. Therefore, the differenced variables in the above regression represent the short run change. We found the differenced variables as being stationary in the data, and we expected the following regression to be free from spuriousness.

$$\Delta \log L_A \quad = \quad k_1 + k_2 \Delta \log A + k_3 \Delta \log \frac{A}{M} + \upsilon_2 \tag{27}$$

We also noted the structural break between the agricultural labour share and the relative TFP in the data. This structural break happened during 1920—1965. We posited 1945 as being the year of structural break and estimated two regressions for eq. (27) for pre- and post-1945 data. We reported the estimation of the structural model (eq. (27)) at the difference level in Table 1. Our estimates show that the change in agricultural labour share for a similar change in either agricultural TFP or relative sectoral TFP is, unsurprisingly, larger in the post-1945 data compared to the till-1945 data.

4.3 Methodology: The Simulated Method of Moments

Our key question concerns what the source of this structural break is. A definite identification strategy is required to identify this structural break as

Independent Variables	Dependent Variable: $\Delta \log L_A$				
	Data			Model	
	Till 1945	Post 1945	All	Till-1945	Post-1945
Constant	**-0.020**	**-0.030**	-0.027	-0.000	-0.001
	(0.008)	(0.007)	(0.005)	(6.2e-4)	(0.010)
$\Delta \log A$	**-0.309**	**-0.421**	-0.281	-0.452	-1.236
	(0.121)	(0.336)	(0.112)	(0.010)	(0.448)
$\Delta \log \frac{A}{M}$	**-0.277**	**-0.388**	-0.060	-0.305	-0.796
	(0.084)	(0.311)	(0.085)	(0.007)	(0.415)
R^2	0.208	0.024	0.076	0.984	0.143
Number of Observation	45	68	113	45	68

Table 1: Structural Model Estimation at the difference

a consequence of the values of the parameters of our model. One possibility for this observed structural break is the evolution of the TFPs for these two sectors. Evaluation of the merit of this possibility requires estimation of the parameters of our model.

We have seven parameters in our model: L, α, σ, β, γ, b, and E. We calibrated all parameters for which testable implications exist. To that effect, we normalised both L and α to unity. This normalisation was done without any loss of generality, as L is a measure of the size of agents and α is the fixed cost to a firm. Among the other parameters of our model, we calibrated σ using the industry markup. Following Hsieh and Klenow [2009, pp-1414], we set the markup for industry at 50% which gives us the value $\sigma = 3$. We carried out a sensitivity analysis by considering $\sigma = 5$ (equivalent to a 25% markup for the industry) without any remarkable change in results.

To tackle the problem of mismatch regarding the absolute level of the relative price and the agricultural labour share, we calibrated the parameters β and γ to match the model's agricultural labour share and the sectoral relative price to their respective empirically observed levels at the initial year of 1820. More specifically, in 1820, the agricultural labour share was at 71.1%, and the relative price between the manufacturing and the agricultural sectors was 1.594. Given a particular set of values for all the other parameters (including b and E), we computed the values of two unknowns– β and γ – by equating the model-predicted agricultural labour share and the relative price to their data counterparts in 1820. The calibration methodology as well the calibrated values of all the parameters are tabulated in Table 2. Understandably, the calibrated values of β and γ depend upon the estimated values of b and E. Therefore, the tabulated values are valid only for our benchmark case.

Parameter	Variable	Method of Calibration	Value
L	Labour force size	Normalisation	1.000
α	Fixed cost of a firm	Normalisation	1.000
σ	Elasticity of substitution in production	Industry Markup	3.000
β	Variable cost	Agricultural labour share in 1820	0.328 (benchmark case)
γ	Subsistence agricultural good consumption	Relative sectoral price in 1820	0.479 (benchmark case)
b	Weight of agricultural good in utility	**Structural model** or agri labour share match	0.723 (Structural model)
E	Elasticity of substitution in utility	**Structural model** or agri labour share match	1.874 (Structural model)

Table 2: Calibration of Parameters

We used the Method of Simulated Moments [Gourieroux and Monfort, 1996] for estimating two remaining key preference parameters: b and E. Using this method, we computed the equilibrium of our model for a given set of values for the parameters. In particular, we calculated the variables, such as the relative sectoral price and the agricultural labour share, from the model for all the periods. We estimated the structural model, as outlined by the regression of eq. (27), using these simulated data from the model. We denoted the estimates of the structural model from the data as Θ^{Data} (tabulated in Data Panel of Table 1) and their counterparts from the simulated data as $\Theta^{Model}(b, E)$. We minimised the distance between these two sets of estimates of the structural model with respect to the two preference parameters, b and E. We measured the distance using a L^2 norm (the sum of the squared differences). We considered those parameter values for which this distance is minimised, as our model estimates.[19] Mathematically,

$$(\hat{b}, \hat{E}) = \arg\min_{\{b,E\}} \left(\Theta^{Data} - \Theta^{Model}(b,E)\right)' \cdot \left(\Theta^{Data} - \Theta^{Model}(b,E)\right) \quad (28)$$

We explained the above procedure with a numerical example. We included k_1, k_2, and k_3 as parameters of our structural model, estimated separately using the till- and post-1945 data. These particular estimates from the data are tabulated in Table 1. We minimised[20] the distance between these six numbers estimated from the data and from the simulated data by changing the parameter values.

While considering different values of the parameter, we imposed the condition that all the parameters must be non-negative. Moreover, we imposed another assumption in the model (discussed in Section 2): $E \leq \sigma$, which im- poses an upper bound for E. The minimum[21] is obtained for the values of $(\hat{b}, \hat{E}) = (0.723, 1.874)$. For this set of values for the parameters, we ran regres-

[19]Numerous applications exist in estimating parameters using the Method of Simulated Moments as we have described here. For a recent such application in labour economics, see Blundell et al. [2016].

[20]We have used discrete state space algorithm as well as simplex method to compute the optimal values.

[21]This minimum value for the distance is given by 0.068.

sion (27) using the simulated data from our model and reported the estimates in Table 1 (Model Panel).

4.4 Alternative Methodology for Estimation: Long Run Labour Share Match

The literature on this topic[22] often measures the performance of the model by its capability to match the time-series of agricultural labour share, observed in the data. In this case, the focus is on long-run evolution of the agricultural labour share rather than any short-run change. We may note that the long run evolution is completely independent of the short-run changes. To compare the performance of our model to the literature, we minimised the distance between agricultural labour shares obtained from the simulated data to their data counterparts. More explicitly, we calculated the difference between agricultural labour share from the data and the simulated data for each period. We minimised the sum of squares of differences by changing the parameter values, as given below:

$$(\hat{b}, \hat{E}) = \arg\min_{\{b,E\}} \sum_{1820 \leq t \leq 2013} \left(\frac{L_{A,t}}{L_t}^{Data} - \frac{L_{A,t}}{L_t}^{Model}(b,E) \right)^2 \tag{29}$$

where $\frac{L_{A,t}}{L_t}^{Data}$ represents the agricultural labour share in the t^{th} period from the data and $\frac{L_{A,t}}{L_t}^{Model}(b, E)$ represents its counterpart in the simulated data. In our calibration, we considered L_t at unity for all periods.

The frequency of data is not same across our time period of investigation, 1820-2013. For the initial years, the data are available at the rate of one in a decade or less, whereas, in the latter half of the sample, the data are accessed annually. If we match the statistics from the data to the model based on

[22] See, for instance, Duarte and Restuccia [2010], Üngör [2017], Bah [2007].

data availability, the latter half of the sample will have unduly more weight. Therefore, we considered the agricultural labour share as a decadal average for the entire time period of our study to calculate our estimates (Table 3). The calibration strategy (Table 2) for the other parameters remains the same, even though the the exact calibrated values of β and γ differ on account of changes in the values of b and E.

Calibration	Estimation Method	Parameters b	E	β	γ
Benchmark	Structural Model	0.723	1.874	0.328	0.479
Benchmark	Labour Share Match	0.742	2.993	0.328	0.417
$\sigma = 5$	Structural Model	0.672	1.978	0.622	0.507
$\sigma = 5$	Labour Share Match	0.725	3.545	0.622	0.415

Table 3: Estimated Values for Model Preference Parameters

5 Discussions

5.1 Benchmark Estimates: Estimation Using the Structural Model

Our results are noted in Table 3. In particular, when we used the structural model (eq. (28)) to estimate our benchmark model, we found $(\hat{b}, \hat{E}) = (0.723, 1.874)$ which demonstrates that consumption of agricultural food beyond the subsistence level has a weight of 72% of the consumption of the industrial good in the consumer's utility function. Moreover, the elasticity of substitution between these two sectoral outputs is more than unity making them substitutes rather than complements. Interestingly, the upper bound of this elasticity (i.e., σ) is not binding for this estimate which is a sign of robustness of our result.

To examine the power of this model in explaining the structural break observed in the data, we compared Figures 4 and 5 to their model counterparts, Figures 6(a) and 6(c). Evidently, our model can generate the structural breaks observed in the data. Since our estimation methodology described in eq. (28) is based on the short-run change of tne agricultural labour share, we used two other moments to evaluate the performance of our model: (1) The agricultural labour share over time and (2) The relative price over time. Figure 6(e) contrasts our model to the data, and we observed that our model can indeed generate the long run diminishing agricultural labour share observed in the data. However, a certain gap remains between the agricultural labour share of this model and its data counterpart throughout most of the times. As far as the relative price is concerned, our model can generate (Figure 6(f)) the initial falling part, and somewhat imperfectly, the latter rising part.

5.1.1 Estimation Using Agricultural Labour Share Match

The other version of our model was estimated using the alternative methodology of period-wise agricultural labour share match described in eq. 29. This version works quite well in matching the agricultural labour share (Figure 7(e)) which is somewhat expected from the method of estimation itself. However, this match emphasises the point that our model performs equally well compared to other models of this literature. This model also can generate structural breaks (Figures 7(a) and 7(c)) though, admittedly, it is not too successful in generating the U-shaped curve of relative price over time observed in the data (Figure 7(f)). The estimate of b changed marginally when we switched to this alternative methodology of agricultural labour share match. However, the estimate of E increased from this change in methodology, and became quite close to its upper boundary of the value of σ when σ was kept at 3.000.

5.1.2 Sensitivity Analysis

We also carried out a sensitivity analysis by calibrating σ at 5.000 for which our estimates are noted in Table 3. Qualitatively, it makes no difference. The power of the new estimates is examined in Figures 8 and 9. An important caveat of this exercise is estimation of E in case of agricultural labour share matching described in eq. 29. The estimate of E is quite close to its upper boundary of the value of σ when σ is kept at 3.000. Once we increased the upper boundary by calibrating σ as 5.000, we observed a change in the estimate of E to 3.545. However, the other conclusions remained either the same or closely similar.

5.2 Identification of Push and Pull channel: A Counterfactual

We discussed in proposition 3 our strategy for identifying the push and the pull channel of the structural change is based on the elasticity of substitution parameter E. The estimated values of E in (all versions) of our model are much larger than unity (Table 3). Hence, we can safely say that "pull" is the dominant channel in our model when we study the aggregated data for the entire period of 1820—2013. Our conclusion coincides with that of Alvarez-Cuadrado and Poschke [2011] for the period before World War I but differs from theirs for the period after World War II, as they concluded that the labour push channel was dominant after World War II.[23]

Our model is somewhat deficient for quantitatively matching the rising relative price observed after World War II. A critique of our model could be that it ignores information embedded in the relative price series that was used by Alvarez-Cuadrado and Poschke [2011] to identify the "push" channel from the "pull" channel. To challenge that critique, we ran the following **counterfactual** to consider the impact of relative price on our conclusion. We considered the estimated values of all parameters from our benchmark estimation except E (Table 2). We estimated the value of E for every period by inputting the actual relative price into our model from the data. In particular, for every time point, we supplied the relative price from data in eq. (12) to derive the variable n which we infused in eq. (15) and solved for E.

Our estimates[24] for the time-series of E show (Figure 10(a)) that both channels worked in different points of the nineteenth century. And, in the twentieth century, the "push" channel was dominant until 1980, and the pull channel has been dominant post-1980. Therefore, our conclusions are quite different from those of Alvarez-Cuadrado and Poschke [2011], even after relative price is taken into account.

Not only are our results different from those presented by Alvarez-Cuadrado and Poschke [2011] but they also from their counterparts derived in our benchmark model, which does not take relative price into account. These disagreements are indicative of the relative price being largely unexplained by the rela-

[23]We quote from their paper: *"...decreases in the relative price of manufactures are unambiguously associated with faster technological change in the nonagricultural sector, i.e., they indicate that the labor pull effect dominates. If the relative price rises, the situation is less clear. An equal proportionate increase in the productivity of both sectors induces an increase in the relative price of manufactures, resulting from the low income elasticity of demand for food and the high-income elasticity of demand for manufactures. So only a strong increase in the relative price is an unambiguous sign of stronger growth in agricultural productivity, or "labor push."..." (pp. 134—135).*

[24]Some of our estimates of E are negative whom we left-censored at a very small number close to zero in the interest of plotting in a log scale.

tive TFP. This could not be detected by Alvarez-Cuadrado and Poschke [2011] as they treated sectoral relative price as a one-to-one function of sectoral TFP (demonstrated as empirically wrong in Figure 4). The falsifiability of our model reveals the problem of prediction using the relative price from the data. In our counterfactual estimates, the agricultural labour share, largely, goes up over time, which does not happen in data (Figure 10(b)). Also, this counterfactual model cannot explain the structural break in the agricultural labour share against the relative TFP, as observed in the data (Figure 10(c)).

Our benchmark falsifiable model, on the other hand, matches both the movement of the relative price and the agricultural labour share to a large extent, which is a testimony to the model's success. (Figures 6(e) and 6(f)) If we set the objective of matching the agricultural labour share to calibrate our model, the model performs at par with other models of this literature. (Figures 7(e) and 7(f))

6 Conclusion

In this paper, we took a fresh look at an old issue concerning the role of sectoral productivities in explaining the structural change of an economy. We built a two-sector general equilibrium model to show that an improvement in agricultural productivity may or may not relocate labour away from the agricultural sector. In fact, an improvement in agricultural productivity can pose a challenge to industrialisation by attracting labour towards the agricultural sector and away from the industrial sector. This possibility occurs in an economy where there is strong substitutability between agricultural and manufacturing goods in preferences and the subsistence food production sector is relatively small in the economy. However, when preferences are complementary, we found a robust relationship between productivity improvement in agriculture and relocation of labour away from this sector. Since, we have seen an historical decline in the share of agricultural labour over time, our theoretical result indicates one of two possibilities: (i) there is substitutability in preference, and productivity improvement in manufacturing is faster than that agriculture (pull channel) and (ii) there is complementarity in preference, and productivity improvement in agriculture is faster than that in manufacturing (push channel).

To identify which one of the above two possibilities is more likely in the data, we looked at the relationship between sectoral relative prices and relative productivities. The data indicated that relative prices are negatively related to relative productivities, although there are structural breaks in these relationships. In a theoretical result developed in the paper (proposition 3), we showed that the relative price (of manufacturing) unambiguously decreases due to a faster productivity improvement in manufacturing. However, when there are faster productivity improvements in agriculture, relative price movements are unpredictable. Therefore, we narrowed down to the first possibility (the pull channel) that results in a declining agricultural labour share along with a declining relative price of manufacturing. In fact, our counterfactual results in the paper highlighted the pull channel as being the dominant factor in explaining the observed data.

Our paper contains several limitations. We modelled a simple static economy without capital. The recent literature shows the importance of capital in a dynamic economy and its substitutability with labour in explaining structural change in an economy. Bringing in learning-by-doing driven growth into our theoretical framework would be an interesting challenge for future work. We have not been able to fully replicate all the empirical patterns in the data through calibration of our theoretical model. Nevertheless, we think that the results in this paper are broadly relevant to understanding the role of sectoral productivities in the process of structural transformation.

References

Alvarez-Cuadrado, Francisco and Markus Poschke, "Structural Change Out of Agriculture: Labor Push versus Labor Pull," *American Economic Journal: Macroeconomics*, 2011, *3* (3), 127 – 58.

__, Ngo Long, and Markus Poschke, "Capital-labor substitution, structural change and growth," *Theoretical Economics*, 2017, *12* (3).

Bah, M. El Hadj, "A Three-Sector Model of Structural Transformation and Economic Development," MPRA Paper 10654, University Library of Munich, Germany 2007.

Baumol, William J., "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis.," *American Economic Review*, 1967, *57* (3), 815–826.

__, Sue Anne Batey Blackman, and Edward N. Wolff, "Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence," *American Economic Review*, 1985, *75* (4), 806–817.

Blundell, Richard, Monica Costa Dias, Costas Meghir, and Jonathan Shaw, "Female labor supply, human capital, and welfare reform," *Econometrica*, 2016, *84* (5), 1705–1753.

Bustos, Paula, Bruno Caprettini, and Jacopo Ponticelli, "Agricultural Productivity and Structural Transformation: Evidence from Brazil," *American Economic Review*, June 2016, *106* (6), 1320 – 1365.

Caselli, Francesco, "Accounting for Cross-Country Income Differences," *Handbook of Economic Growth*, 2005, *1, Part A*, 679 – 741.

Craig, Lee A. and Thomas Weiss, "Hours at work and total factor productivity growth in nineteenth-century U.S. agriculture," in "New Frontiers in Agricultural History" 2000, pp. 1–30.

Dennis, Benjamin N and Talan B İşcan, "Engel versus Baumol: Accounting for structural change using two centuries of US data," *Explorations in Economic History*, 2009, *46* (2), 186–202.

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," *Quarterly Journal of Economics*, 2010, *125* (1), 129 – 173.

Duranton, Gilles, "Agricultural productivity, trade, and industrialisation," *Oxford Economic Papers*, 1998, *50* (2), 220 – 236.

Field, Alexander James, "Sectoral shift in antebellum Massachusetts: A reconsideration," *Explorations in Economic History*, 1978, *15* (2), 146 – 171.

Foster, Andrew D. and Mark R. Rosenzweig, "Agricultural Productivity Growth, Rural Economic Diversity, and Economic Reforms: India, 1970-2000," *Economic Development and Cultural Change*, 2004, *52* (3), 509 – 542.

__ and , "Economic Development and the Decline of Agricultural Employment," *Handbook of Development Economics*, 2007, *4*, 3051 – 3083.

Gallman, Robert E., "Changes in Total U.S. Agricultural Factor Productivity in the Nineteenth Century," *Agricultural History*, 1972, *46* (1), 191–210.

Gollin, Douglas and Richard Rogerson, "Productivity, transport costs and subsistence agriculture," *Journal of Development Economics*, 2014, *107*, 38 – 48.

__, Remi Jedwab, and Dietrich Vollrath, "Urbanization with and without industrialization," *Journal of Economic Growth*, 2016, *21* (1), 35 – 70.

__, Stephen L. Parente, and Richard Rogerson, "The food problem and the evolution of international income levels," *Journal of Monetary Economics*, 2007, *54* (4), 1230 – 1255.

__, Stephen Parente, and Richard Rogerson, "The Role of Agriculture in Development," *American Economic Review*, 2002, *92* (2), 160 – 164.

Gourieroux, Christian and Alain Monfort, *Simulation-based econometric methods*, Oxford university press, 1996.

Henderson, J. Vernon, Roberts Mark, and Storeygard Adam, "Is urbanization in Sub-Saharan Africa different?," *Policy Research working paper, Washington, DC: World Bank*, 2013, *no. WPS 6481*, 60 – 82. Washington, DC: World Bank.

__, Storeygard Adam, and Deichmann Uwe, "Has climate change driven urbanization in Africa?," *Journal of Development Economics*, 2017, *124*, 60 – 82.

Herrendorf, Berthold and Ãkos Valentinyi, "Which Sector Make Poor Countries so Unproductive?," *Journal of the European Economic Association*, 2012, *10* (2), 323 – 341.

__, Richard Rogerson, and Ãkos Valentinyi, "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, December 2013, *103* (7), 2752 – 89.

__, , and , "Growth and Structural Transformation," *Handbook of Economic Growth*, 2014, *2*, 855 – 941.

Hsieh, Chang-Tai and Peter J Klenow, "Misallocation and manufacturing TFP in China and India," *The quarterly journal of economics*, 2009, *124* (4), 1403–1448.

Jedwab, Remi, "Urbanization without Structural Transformation: Evidence from Consumption Cities in Africa," 2013. Working paper.

Kendrick, John W, “Front matter, Productivity Trends in the United States,” in “Productivity Trends in the United States,” Princeton University Press, 1961, pp. 52–0.

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie, “Beyond Balanced Growth,” *Review of Economic Studies*, 2001, *68* (4), 869 – 882.

Matsuyama, Kiminori, “Agricultural productivity, comparative advantage, and economic growth,” *Journal of Economic Theory*, 1992, *58* (2), 317 – 334.

Mokyr, Joel, *Industrialization in the Low Countries, 1795-1850*, Yale University Press, 1977.

__, “The industrial revolution and the Netherlands: Why did it not happen?,” *De Economist*, 2000, *148* (4), 503 – 520.

Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny, “Industrialization and the Big Push,” *Journal of Political Economy*, 1989, *97* (5), 1003 – 1026.

Ngai, L. Rachel and Christopher A. Pissarides, “Structural Change in a Multisector Model of Growth,” *American Economic Review*, 2007, *97* (1), 429 – 443.

Popper, Karl, *The logic of scientific discovery*, Routledge, 2005.

Ragnar, Nurkse, *Problems of Capital Formation in Underdeveloped Countries*, New York: Oxford Press. University, 1953.

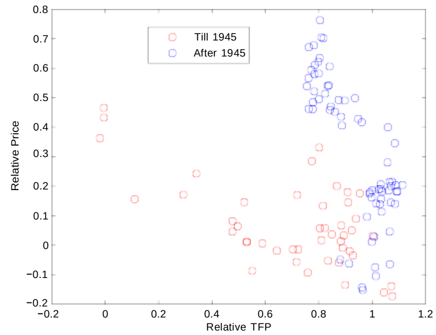
Rostow, W. W., *The Stages of Economic Growth: A Non-Communist Manifesto*, Cambridge University Press, 1960.

Schultz, Theodore W., *The Economic Organization of Agriculture*, New York: McGraw-Hill, 1953.

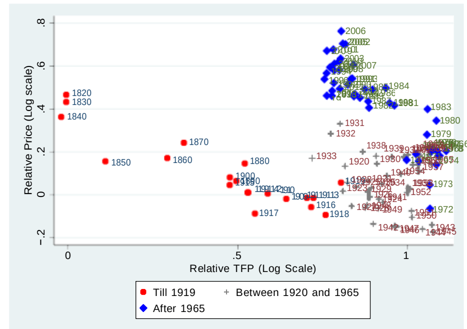
Sokoloff, Kenneth L, “Productivity growth in manufacturing during early industrialization: Evidence from the American northeast, 1820-1860,” in “Long-term factors in American economic growth,” University of Chicago Press, 1986, pp. 679–736.

Üngör, Murat, “Productivity growth and labor reallocation: Latin America versus East Asia,” *Review of Economic Dynamics*, 2017, *24*, 25–42.

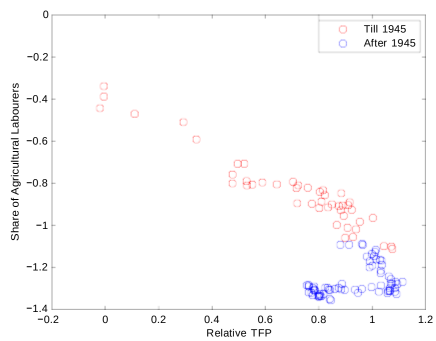
Wright, Gavin, “Cheap Labor and Southern Textiles before 1880,” *The Journal of Economic History*, 1979, *39* (3), 655 – 680.



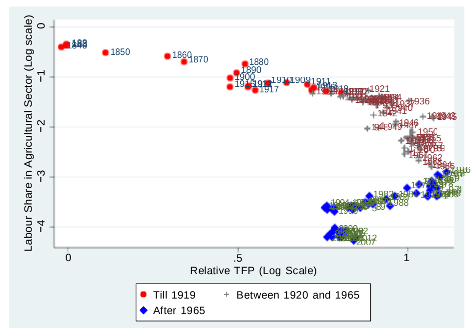
(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



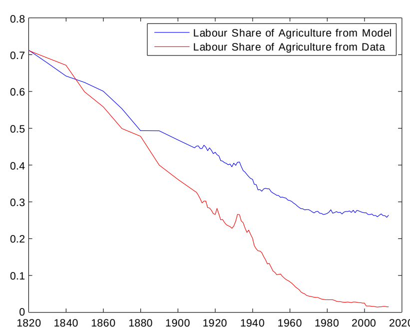
(b) Relative price against Relative TFP (in logarithmic scale): Data contrast till 1945 to post-1945



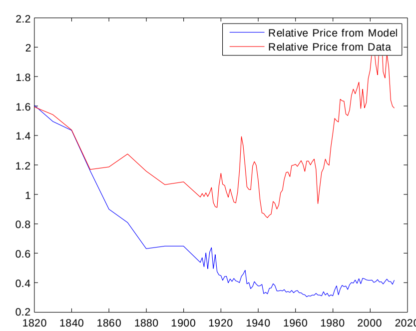
(c) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)



(d) Agricultural Labour Share against Relative TFP (in logarithmic scale: Data contrasts till 1945 to post-1945)



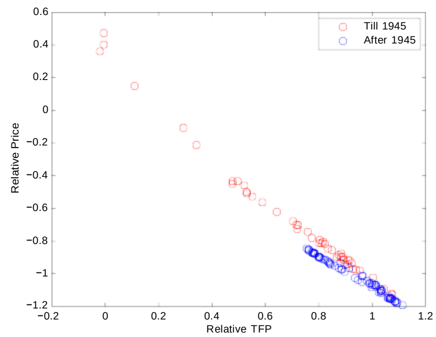
(e) Agricultural Sector Labour Share during 1820–2013: Model versus Data

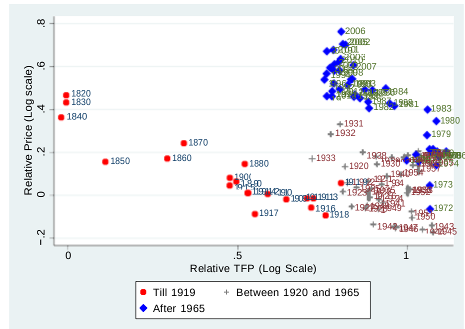


(f) Relative Price during 1820–2013: Model versus Data

Figure 6: The estimated benchmark (eq. (28)) values of the parameters are: $(\hat{b}, \hat{E}) = (0.723, 1.874)$. They were obtained by minimising the distance between the structural model (regression eq. (27)) estimates obtained from the data

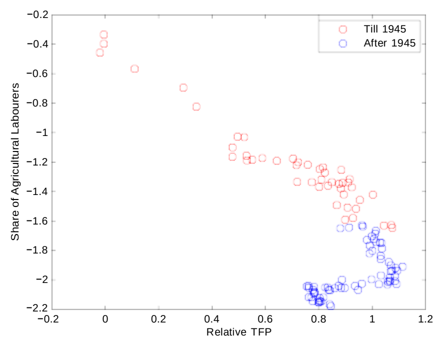
and their counterparts estimated from the simulated data. Figures (a) and (c) are from our model whose data counterparts are presented as Figures (b) and (d), to demonstrate the structural break. Figures (e) and (f) contrast the model with the data regarding agricultural labour share and relative sectoral price over time.

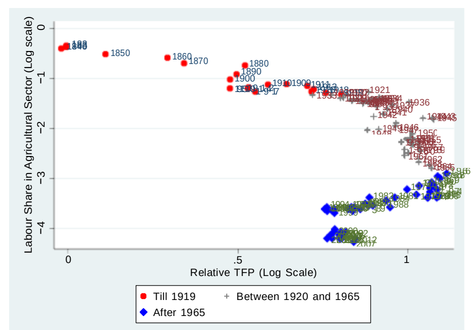




(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945

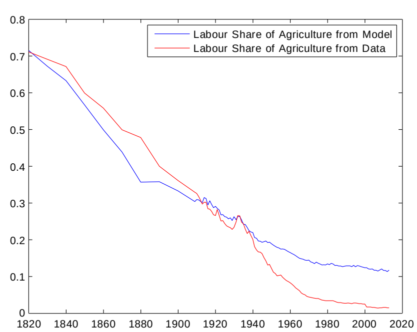
(b) Relative price against Relative TFP (in logarithmic scale): Data contrast till 1945 to post-1945

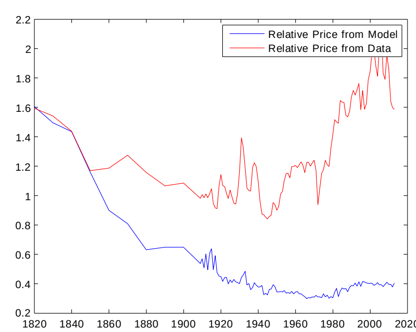




(c) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

(d) Agricultural Labour Share against Relative TFP (in logarithmic scale: Data contrasts till 1945 to post-1945)

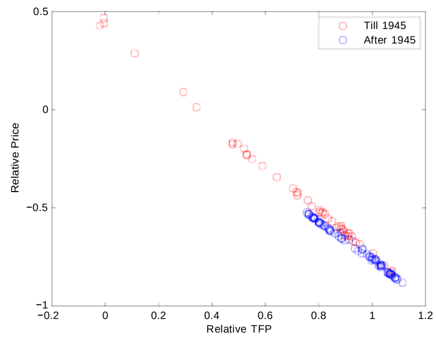




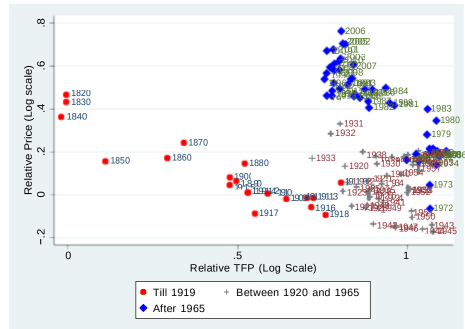
(e) Agricultural Sector Labour Share during 1820–2013: Model versus Data

(f) Relative Price during 1820–2013: Model versus Data

Figure 7: The estimated (eq. (29)) values of the parameters are: $(\hat{b}, \hat{E}) = (0.742, 2.993)$. They were estimated by minimising the distance between period-wise agricultural labour share obtained from the data and their counterparts obtained from the simulated data. Figures (a) and (c) are from our model whose data counterparts are presented as Figures (b) and (d), to demonstrate the structural break. Figures (e) and (f) contrast the model with the data regarding agricultural labour share and relative sectoral price over time.



(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



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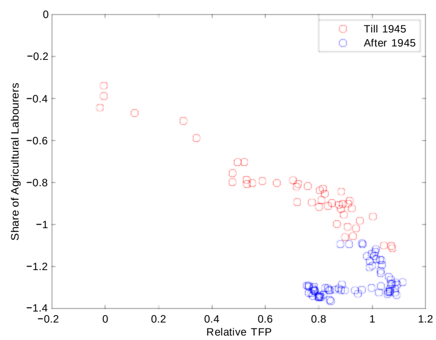

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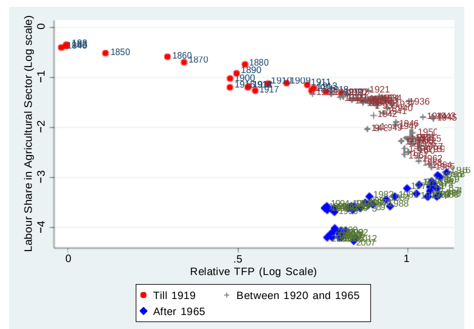
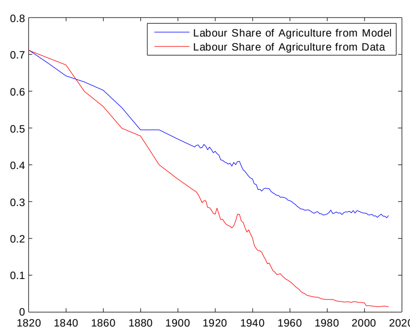
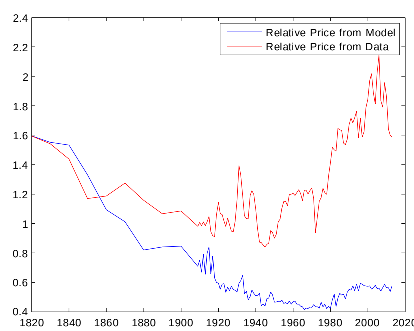


(d) Agricultural Labour Share against Relative TFP (in logarithmic scale: Data contrasts till 1945 to post-1945)

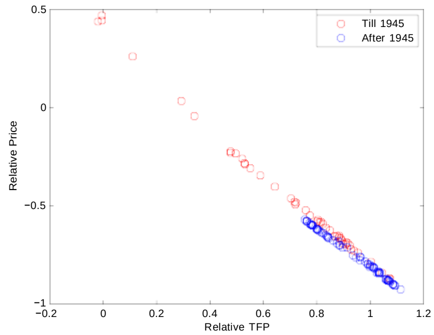


(e) Agricultural Sector Labour Share during 1820–2013: Model versus Data

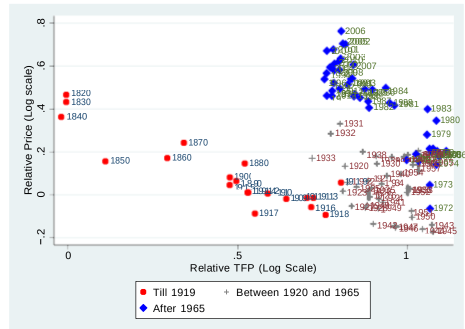


(f) Relative Price during 1820–2013: Model versus Data

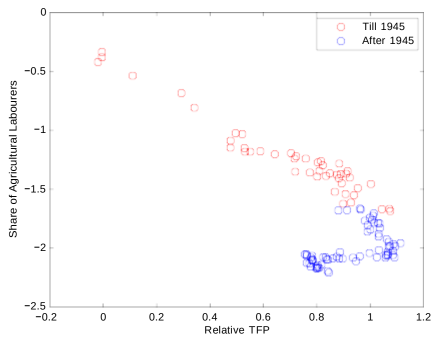
Figure 8: The estimated benchmark (eq. (28)) values of the parameters are: $(\hat{b}, \hat{E}) = (0.672, 1.978)$, given σ is calibrated at 5.000. They were obtained by minimising the distance between the structural model (regression eq. (27)) estimates obtained from the data and their counterparts estimated from the simulated data. Figures (a) and (c) are from our model whose data counterparts are presented as Figures (b) and (d), to demonstrate the structural break. Figures (e) and (f) contrast the model with the data regarding agricultural labour share and relative sectoral price over time.



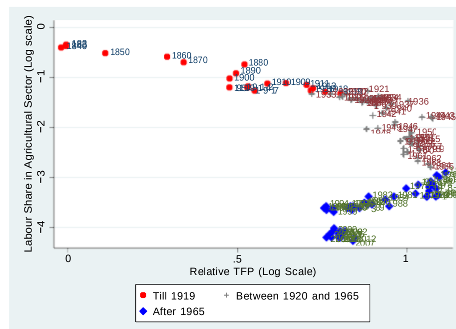
(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



(b) Relative price against Relative TFP (in logarithmic scale): Data contrast till 1945 to post-1945



(c) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)



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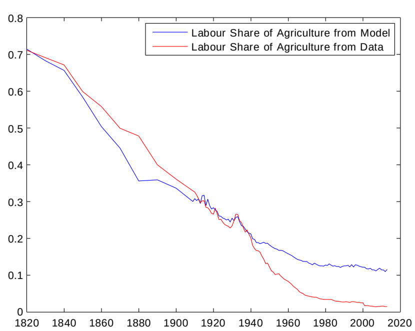
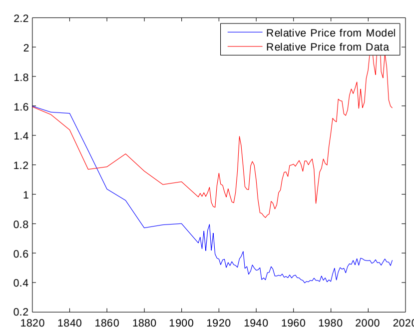
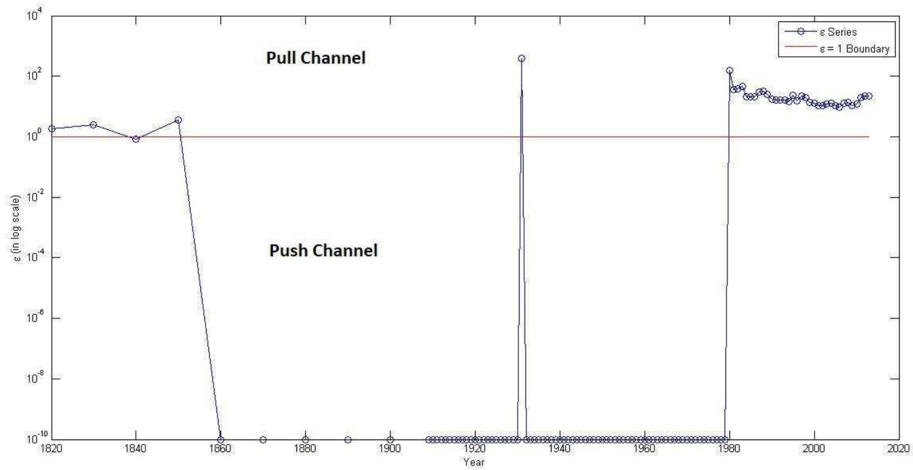


(e) Agricultural Sector Labour Share during 1820–2013: Model versus Data

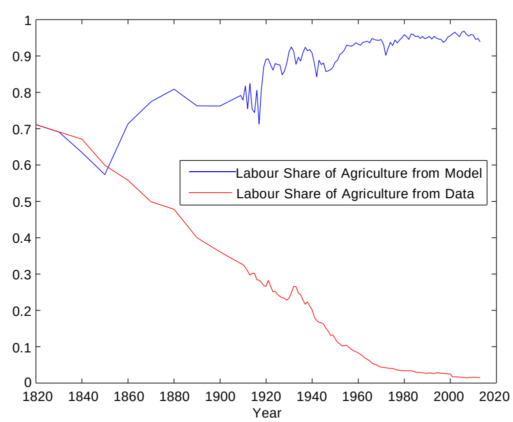


(f) Relative Price during 1820–2013: Model versus Data

Figure 9: The estimated (eq. (29)) values of the parameters are: $(\hat{b}, \hat{E}) = (0.725, 3.545)$, given σ is calibrated at 5.000. They were estimated by minimising the distance between period-wise agricultural labour share obtained from the data and their counterparts obtained from the simulated data. Figures (a) and (c) are from our model whose data counterparts are presented as Figures (b) and (d), to demonstrate the structural break. Figures (e) and (f) contrast the model with the data regarding agricultural labour share and relative sectoral price over time.



(a) Counterfactual: E estimates over during 1820–2013: Push versus Pull channels



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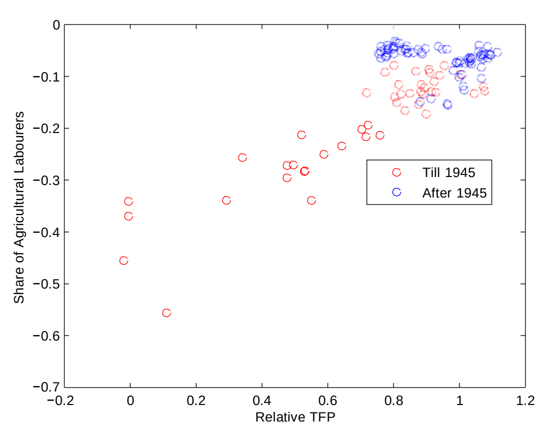

(c) Relative Price during against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

Figure 10: A counterfactual with benchmark values for other parameters (σ = 3, b = 0.723, β = 0.328, γ = 0.479) was run in which the relative price from the data was infused to estimate E at each point in time. E = 1 line is the boundary

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